STOCHASTIC PARTICLE ACCELERATION IN SOLAR FLARES

W. Dröge, R. Schlickeiser
Max-Planck-Institut für Radioastronomie
Auf dem Hügel 69, 5300 Bonn 1, FRG

ABSTRACT

We propose that particles during the second phase of solar flares are accelerated by stochastic resonant scattering off hydromagnetic waves and first-order Fermi acceleration in shock waves generated in the impulsive phase of the flare. Our solutions allow arbitrary power law momentum dependences of the momentum diffusion coefficient as well as the momentum loss time. The acceleration time scale to a characteristic energy $\sim$100 keV for protons can be as short as 5 s. The resulting electron spectra show a characteristic double power law with a transition around 200 keV and are correlated to the proton spectra evaluated under equal boundary conditions, indicating that electrons and protons are accelerated by the same mechanism. The correlation between the different spectral indices in the electron double power law and between electron and proton spectra are governed by the ratio of first-to second-order acceleration and therefore allow a determination of the Alfvén Mach number of the shock wave.

1. Introduction. We propose that the combination of first- and second-order Fermi acceleration is the mechanism responsible for second-phase acceleration in solar flares. This model has a number of distinct advantages over previous models, in that it gives a natural explanation for the various observed time delays between first and second phase as well as for the dependence of the particle spectra to the strength of the flare and the correlation between electron and proton spectral indices.

2. Theory. Parker and Tidman (1958) have pointed out that Fermi acceleration is an intrinsic property of any sufficiently agitated plasma of energetic particles. The effects of second-order Fermi acceleration due to irregularly moving magnetized fluid elements, first-order Fermi acceleration off strong shocks as well as loss and escape processes can be incorporated into a transport equation in phase space (Ramaty 1979)

$$\frac{\partial f}{\partial t} - \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D(p)\frac{\partial f}{\partial p}\right] - \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 (\dot{p}_G + \dot{p}_L) f\right] + \frac{f}{T(p)} = Q(p,t) \qquad (1),$$

where p is the particle momentum, $N(p) = 4\pi p^2 f(p,t)$ the number of particles per unit momentum and unit volume, and $Q(p,t)$ represents sources and sinks of particles. Using the concept of the age distribution (Schlickeiser and Lerche 1985) the effects of spatial diffusion, convection and catastrophic losses have been combined in a momentum-dependent escape time $T(p) = T_o\, p^{-b}$.

In a plasma with a strong MHD turbulence both the momentum diffusion coefficient $D(p)$ and the spatial diffusion coefficient along the mean magnetic field $K_{\parallel}(p)$ are governed by the turbulence simultaneously. By using quasi-linear theory $K_{\parallel}(p)$ can be related rigorously to the spectrum

of the magnetic field fluctuations (Jokipii 1977). Assuming a power law spectrum for the magnetic irregularities $W(k) = W_o\, k^{-q}$ we may write

$$K_{\parallel}(p) = \delta\, v(p)\, p^{2-q} = \kappa p^{\eta} \qquad (2)$$

(δ,κ = const.) and

$$D(p) = \alpha_2 \frac{V_A^2}{K_{\parallel}} p^2 \qquad (3)$$

(V_A: Alfvén speed, α_2: constant). As gain process, we consider quasi-continuous momentum gain by acceleration at a shock wave moving through the plasma with speed V_S. The momentum gain by first-order Fermi acceleration at a single shock wave has been determined by Drury (1983)

$$\dot{p}_G = \alpha_1 \frac{V_S^2}{K_{\parallel}} p \qquad (4)$$

(α_1: constant). Möbius et al. (1982) pointed out that for conditions in the flare site the rate of systematic acceleration exceeds the rate of momentum loss. Thus we assume that $\dot{p}_G \gg \dot{p}_L$ at least at all momenta of interest. Using (2), (3) and (4) in equation (1) and considering the steady-state case ($\partial f/\partial t = 0$) yields

$$\frac{1}{p^2} \frac{d}{dp} \left[p^{4-\eta} \frac{df}{dp} - a\, p^{3-\eta}\right] - \lambda\, p^b = Q(p) \qquad (5)$$

($a_1 = \alpha_1 V_S^2/\kappa$, $a_2 = \alpha_2 V_A^2/\kappa$, $a = a_1/a_2$, $\lambda = 1/a_2 T_Q$). We assume that the injection takes place at some momentum $Q(p) = q_o\, \delta(p-p_o)$ and we find for the steady-state particle number density $N(p) = 4\pi p^2 f(p)$ (see Dröge and Schlickeiser (1985) for details):

$$N(p) = \frac{8\pi q_o}{a_2 |\eta+b|}\, p_o^{\frac{\eta-a+1}{2}}\, p^{\frac{\eta+a+1}{2}} \begin{cases} K_\nu\left(\frac{2\lambda^{1/2}}{|\eta+b|} p_o^{\frac{\eta+b}{2}}\right) I_\nu\left(\frac{2\lambda^{1/2}}{|\eta+b|} p^{\frac{\eta+b}{2}}\right) & p < p_o \\ I_\nu\left(\frac{2\lambda^{1/2}}{|\eta+b|} p_o^{\frac{\eta+b}{2}}\right) K_\nu\left(\frac{2\lambda^{1/2}}{|\eta+b|} p^{\frac{\eta+b}{2}}\right) & p > p_o \end{cases} \qquad (6)$$

$\nu = |(3+a-\eta)/(\eta+b)|$. In the limit $\eta+b \to 0$ solution (6) approaches a power law distribution

$$N(p) = \frac{2\pi q_o}{a_2\sqrt{\frac{(3+a-\eta)^2}{4}+\lambda}}\, p_o^{\frac{\eta-a+1}{2}}\, p^{\frac{\eta+a+1}{2}} \begin{cases} (p/p_o)^{\sqrt{\frac{(3+a-\eta)^2}{4}+\lambda}} & p < p_o \\ (p/p_o)^{-\sqrt{\frac{(3+a-\eta)^2}{4}+\lambda}} & p > p_o \end{cases} \qquad (7)$$

Equations (6) and (7) are generalizations of the solutions of Ramaty (1979; $b=0$, $a=0$, $\eta=0,1$), Barbosa (1979; $a=0$), Pikel'ner and Tsytovich (1976; $a=0$, $b=0$). The solutions (6) and (7) allow arbitrary momentum power law dependences of the spatial diffusion coefficient (η) and the escape time (b). The parameter $a = (\alpha_1/\alpha_2)(V_S^2/V_A^2) = (\alpha_1/\alpha_2)\, M_A^2$ is of the order of the square of the shock's Mach number and describes the ratio of first- to second-order Fermi acceleration. In the case of no Fermi ac-

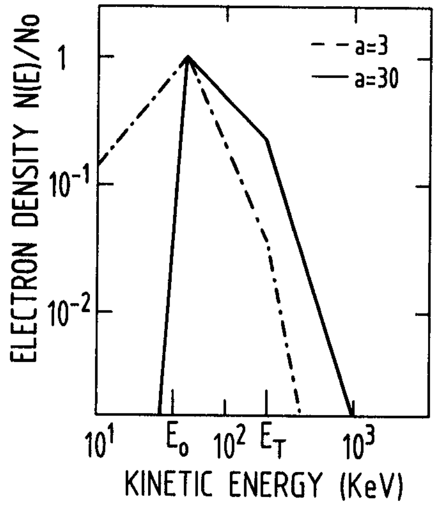

Fig. 1: Normalized steady-state electron number density. Particles are injected at $E_o = 50$ keV. At $E_T = 200$ keV a transition from solution (6) to (7) occurs.

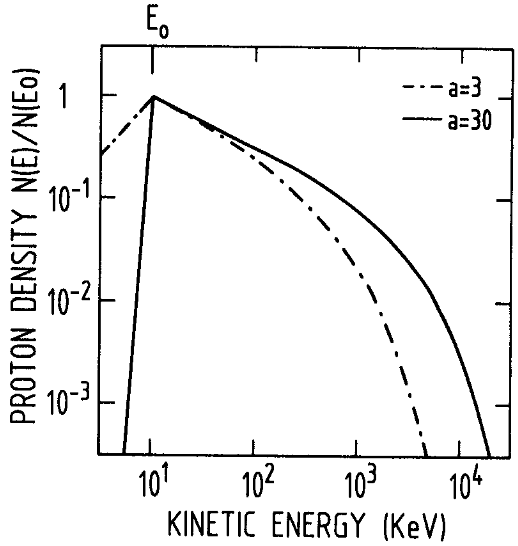

Fig. 2: Normalized steady-state proton number density. Particles are injected at $E_o = 10$ keV.

celeration at shock waves ($a = 0$) (6) and (7) reduce to the solutions of previous models as quoted above.

3. Discussion. The energy spectra of electrons accelerated in large flares exhibit a characteristic double power law with a break around $E_T \cong 200$ keV (Lin et al. 1982). From this we conclude that for nonrelativistic kinetic energies ($E < E_T$) the Bessel function solution (6) holds, which for small arguments is approximately a power law, whereas for $E > E_T$ the particles are relativistic ($p = mc \mathrel{\hat{=}} E = 200$ keV for electrons) and the distribution function approaches the power law solution (7) (cf. Fig. 1). We take spatial diffusion along the mean magnetic field as the relevant escape process. The escape time then is $T(p) = L^2/K_\parallel = (L^2/\kappa)\ p^{-\eta}$, where L is the length scale of the system and $\eta = b = 0$ in the relativistic and $\eta = b = 1$ in the nonrelativistic case. Thus we may consider the evolution of the particle spectra in the hard-sphere approximation (cf. Möbius et al. 1982), which gives us

$$K_\parallel(p) \quad = \quad \frac{1}{3}\, v(p)\, \ell \tag{8}$$

$v(p)$ is the velocity and ℓ the momentum independent mean free path of the particles. For comparison with data we transform (6) and (7) into energy space where $N(E) = N(p)\ (1/v)$ and $E = (p^2c^2+m^2c^4)^{1/2} - mc^2$ is the particle kinetic energy. An "effective power law" exponent can be calculated ($E > E_o$)

$$\gamma_{eff}(E) = -\frac{d\,\lg N(E)}{d\,\lg E} = \begin{cases} -\dfrac{a}{4} + \dfrac{a+2}{4}\sqrt{1+\dfrac{4x^2}{(a+2)^2}\dfrac{2E}{mc^2}} & p < mc \qquad (9) \\[2ex] -\dfrac{a+1}{2} + \dfrac{a+3}{2}\sqrt{1+\dfrac{4x^2}{(a+3)^2}} & p > mc \qquad (10) \end{cases}$$

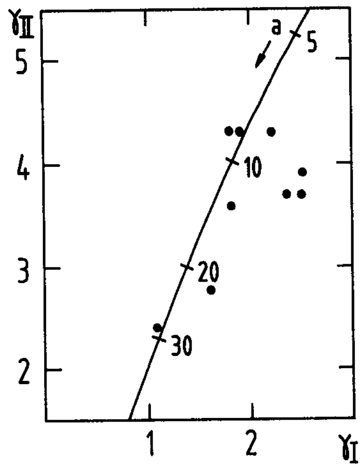

Fig. 3: Correlation between electron spectral exponent below 200 keV γ_I and exponent above 200 keV γ_{II} as a function of a

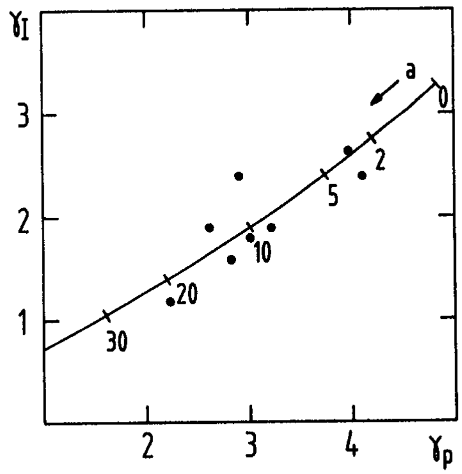

Fig. 4: Correlation between γ_I and proton spectral exponent above 10 MeV as a function of a

$x = \ell_c/L\,V_A$ is a free parameter which may have different values for electrons and protons and for different flares. Equations (9) and (10) allow us to determine the correlation between the mean spectral index below 200 keV γ_I and above 200 keV γ_{II} for electrons (Fig. 3) as well as the correlation between γ_I and the proton spectral exponent above 10 MeV γ_p (Fig. 4) as a function of a. The curves obtained from (9) and (10) are in excellent agreement with the measurements of Lin et al. (1982), indicating that the strongest shock waves have an Alfvén Mach number $M_A \sim 4$, corresponding to $a \sim 30$ (see Dröge and Schlickeiser (1985) for details).

4. Conclusions. Combining first- and second-order Fermi mechanism in solar flare second-phase acceleration successfully explains the observed ion and electron energy spectra. The model correctly accounts for the sometimes very short delay times, and reproduces quantitatively the correlations of nonrelativistic with relativistic electron spectral indices, and the correlation of nonrelativistic electron with proton indices.

References

Barbosa, D.D., (1979), Astrophys. J. 233, 383
Dröge, W., Schlickeiser, R., (1985), in preparation
Drury, L.O.C., (1983), Rept. Progr. Phys. 46, 973
Jokipii, J.R., (1977), Proc. 15th Int. Cosmic Ray Conf. (Plovdiv), Vol. 2, 429
Lin, R.P. et al., (1982), Astrophys. J. 253, 949
Möbius, E. et al., (1982), Astrophys. J. 259, 397
Parker, E.N., Tidman, D.A., (1958), Phys. Rev. 111, 1206
Pikel'ner, S.B., Tsytovich, V.N., (1976), Sov. Astron. 19, 450
Ramaty, R., (1979), Particle Acceleration Mechanisms in Astrophysics, eds. J. Arons, C. Max, C. McKee, American Institute of Physics, New York, p. 135
Schlickeiser, R., Lerche, I., (1985), this conference, OG 7.2-8